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IMAGE DEBLURRING WITH FILTERING APPROACH

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ABSTARCT:

This paper is based on image deblurring with filtering approach. Wiener filter is a method giving the best results when variance of the noise incorporated in blurring process is known as a priori. The paper also discusses basis blurring forms and their mathematical description. Image deblurring refer to procedures that attempt to reduce the blur amount in a blurry image and grant the degraded image an overall sharpened appearance to obtain a clearer image. The point spread function (PSF) is one of the essential factors that needed to be calculated, since it will be employed with different types of deblurring algorithms and filters approaches.

Keywords: Image deblurring techniques, blur, image degradations, point spread function (PSF), image restoration, iterative techniques, Wiener filter.

1. INTRODUCTION

Image deblurring plays an important role in an image restoration process. Image capture process causes degradation of original image. There are several factors having contribution to blurring, two of them are the most important [1]:

• Movement of camera or capturing object when long exposure time is set, being called motion blur.

• Out of focus optic caused by wide angle lens setting or atmospheric turbulence, being called out of focus blur degraded image is additionally corrupted by the noise. The noise is the consequence of imperfection of image sensor and acquisition part of camera. Degradation process can be described by the following formula [2]:

g = kf + n(1)

Where : g is a vector corresponding to blurred (degraded) image, K is a large usually ill conditioned matrix modeling blurring operation, f is vector corresponding to perfect image and n is the noise vector. Degradation process can also be presented in another form [1]:

$$g(n_1, n_2) = k(n_1, n_2) * f(n_1, n_2) + n(n_1, n_2)$$

= $\sum_{k_1}^{N-1} \sum_{k_2}^{M-1} k(k_1, k_2) f(n_1 - k_1, n_2 - k_2) + n(n_1, n_2)$ (2)

where: g(n1, n2) is blurred (degraded) image, k(n1, n2) is kernel or point-spread function (PSF) modeling blurring operation, f(n1, n2) is perfect image, n(n1, n2) is the noise, N and M correspond to the number of image pixels in horizontal and vertical axes respectively and asterisk (*) stands for convolution.



2. PREVIOUS WORK

a. Image Deblurring – Wiener Filter Versus TSVD Approach paper by P. Bojarczak and Z. Łukasik has introduce the working with performance comparison of Wiener Filter and TSVD Approach. Wiener filter is a method giving the best results when variance of the noise incorporated in blurring process is known a priori. In TSVD decomposition the knowledge of precise variance of the noise is not necessary to image restoration. The paper also discusses basis blurring forms and their mathematical description. TSVD method has an advantage allowing for the estimation noise level of the image on the basis of Picard plot, what makes it attractive in application where the information about noise is not available a priori. On the other hand w hen the detailed information about noise level of image is well known, then Wiener filter seems to be

a better solution.

b. High-quality Motion Deblurring from a Single Image paper by Qi Shan, Jiaya Jia and Aseem Agarwala has present a new algorithm for removing motion blur from a single image. Our method computes a deblurred image using a unified probabilistic model of both blur kernel estimation and unblurred image restoration. present an analysis of the causes of common artifacts found in current deblurring methods, and then introduce several novel terms within this probabilistic model that are inspired by our analysis. These terms include a model of the spatial randomness of noise in the blurred image, as well a new local smoothness prior that reduces ringing artifacts by constraining contrast in the unblurred image wherever the blurred image exhibits low contrast. Finally, describe an efficient optimization scheme that alternates between blur kernel estimation and unblurred image restoration until convergence. As a result of these steps, able to produce high quality deblurred results in low computation time. Also able to produce results of comparable quality to techniques that require additional input images beyond a single blurry photograph, and to methods that require additional hardware.

c. Rotational Motion Deblurring of a Rigid Object from a Single Image paper by Qi Shan, Wei Xiong, and Jiaya Jia has present : Most previous motion deblurring methods restore the degraded image assuming a shift-invariant linear blur filter . These methods are not applicable if the blur is caused by spatially variant motions. In this paper, model the physical properties of a 2-D rigid body movement and propose a practical framework to deblur rotational motions from a single image. The main observation is that the transparency cue of a blurred object, which represents the motion blur formation from an imaging perspective, provides sufficient information in determining the object movements. Comparatively, single image motion deblurring using pixel color/gradient information has large uncertainties in motion representation and computation. The effectiveness of method is demonstrated using challenging image examples.

d. Single Image Motion Deblurring Using Transparency by Jiaya Jia, The Chinese University of Hong Kong 2007 IEEE paper has demonstrate :One of the key problems of restoring a degraded image from motion blur is the estimation of the unknown shifting variant linear blur filter . Several algorithms have been proposed using image intensity or gradient information. In this paper , separate the image deblurring into filter estimation and image deconvolution processes, and propose a novel algorithm to estimate the motion blur filter from a perspective of alpha values. The relationship between the object boundary transparency and the image motion blur is in vestigated, formulate the filter estimation as solving a Maximum a Posteriori (MAP) problem with the defined likelihood and prior on transparency.

3. DEBLURRING OF IMAGE WITH WIENER FILTER:

The Wiener filter is used for deblurring an image in the case when the blur kernel (point spread function) is known. We compute the Fourier transform of the deblurred image F

by: F = (G / H) * (|H| 2 / (k+|H| 2))

where

G = Fourier transform of original blurry image

- H = Fourier transform of blur kernel
- $k = deblurring \ parameter \ (k \ge 0)$

Setting the value of k is the hard part. The value should depend on the amount of noise we expect in the image. If the M x N image has Gaussian white noise with variance σ^2 then we would set $k = MN\sigma^2$. But in general, we



don't know the variance of the noise so we will have to estimate it. Note that if the image has no noise (blur only), then we can set k=0.

The inverse filter does a terrible job due to the fact that it divides in the frequency domain by numbers that are very small, which amplifies any observation noise in the image. In this blog, I'll look at a better approach, based on the Wiener filter.

If the blurring process is presented in the form (2), then the restored image can be obtained by convoluting with the PSF function of linear filter [1]

$$\hat{f}(n_1, n_2) = h(n_1, n_2) * g(n_1, n_2)$$
$$= \sum_{k_1}^{N-1} \sum_{k_2}^{M-1} h(k_1, k_2) g(n_1 - k_1, n_2 - k_2)$$

Or in spectral domain:

$$\hat{F}(u,v) = H(u,v)G(u,v)$$
where: $\hat{F}(u,v) = DFT2\left[\hat{f}(n_1,n_2)\right],$

$$H(u,v) = DFT2\left[h(n_1,n_2)\right],$$

 $G(u,v) = DFT2[g(n_1,n_2)]$ and DFT2 means two dimensional Discrete Fourier Transform. When noise term in (2) is absent, then the relationship in spatial domain between blurring PSF function and PSF function of linear filter is of the form:

$$h(n_1, n_2) * k(n_1, n_2)$$

= $\sum_{k_1}^{N-1} \sum_{k_2}^{M-1} h(k_1, k_2) k(n_1 - k_1, n_2 - k_2) = \delta(n_1, n_2)$

 $\delta(n_1, n_2)$ is Dirac delta function and in spectral domain:

$$H(u,v)K(u,v) = 1 \Longrightarrow H(u,v) = \frac{1}{K(u,v)}$$
$$K(u,v) = DFT2[k(n_1,n_2)]$$

In this case, the blurred image can be perfectly reconstructed by linear filter of PSF function fulfilling the relationship (9). However when noise term in (3) is present, then application of filter of fulfilling the relationship (9) leads to the following reconstruction of degraded image [1]:

$$\stackrel{\Lambda}{F}(u,v) = H(u,v)G(u,v)$$
$$= \frac{1}{K(u,v)} (K(u,v)F(u,v) + N(u,v))$$
$$= F(u,v) + \frac{N(u,v)}{K(u,v)}$$
$$N(u,v) = DFT2[n(n_1,n_2)]$$

As it can be seen from (10), two factors can cause poor reconstruction of the image. First resulting from existence of frequencies (u, v), for which K(u, v) approaches to zero, which in turn can lead to

IJRAT

the lack of the solution of (10). Second even if the solution happens to exist, for frequencies (u, v), for which K(u, v) has small values, the noise term is amplified significantly second term in (10). In order to minimize the influence of noise term on the whole reconstruction, the PSF function of filter is calculated such that minimizes the mean-squared error (MSE) between original $f(n \ 1 \ n2)$ and reconstructed image $f^{(n)}(n \ 1 \ n2)$

$$MSE = \sum_{N_1}^{N-1} \sum_{N_2}^{M-1} \left(f(n_1, n_2) - f(n_1, n_2) \right)^2$$

Minimization of (11) leads to modified formula for h(n1,n2) PSF function of filter having following for min spectral domain

$$H(u,v) = \frac{K^{*}(u,v)}{K^{*}(u,v)K(u,v) + [S_{n}(u,v)/S_{f}(u,v)]}$$

where: $k^*(u, v)$ is a complex conjugate of k(u, v) and $s_n(u, v)$ and $s_f(u, v)$ are the power spectrum of the noise and ideal image respectively. If the noise is uncorrelated then its power spectrum is easy available by:

$$S_N(u,v) = \sigma_n^2$$
 for all (u,v)
Where: σ_n^2 is a noise variance.

Based on it, it is possible to estimate power spectrum of original image on the basis of power spectrum of blurred image, $s_g(u, v)$ and the information about the noise variance [1].

$$S_f(u,v) = S_g(u,v) - \boldsymbol{\sigma}_n^2$$



a.



b.



Fig1. Image deblurring process by wiener filter

In Wiener filter approach, information about the noise variance is necessary to good quality restoration. Fig.1a shows original image, and Fig.1b shows distorted image through out of focus blur of L=10 with Gaussian noise of variance=1. Fig1c shows the image restored by Wiener filter tuned to variance of 1. Fig1d shows image restored by the same filter with the variance parameter of 5. As it can be seen, Wiener filter tuned to the actual noise variance almost perfectly reconstructs the original image (INSR=1,55dB). However, mismatching filter variance parameter to actual noise variance causes significant degradation of filter's performance (INSR=-0.13dB).

The main drawback of Wiener filter is the necessity of a priori knowledge of type and magnitude of noise, which is often unavailable or hardly accessible in practice.

4. FOURIER FILTERING

4.1 INTRODUCTION & OBJECTIVES

Almost every system or signal that scientists and engineers deal with can be viewed in several different ways. They can exist as a function of space, defined by physical parameters like length, width, height, color intensity, and others. They can exist as a function of time, defined by changes in any measurable characteristic. They can also exist as a function of frequency, defined by the composition of periodicity that make up light, sound, space, or any other dynamic system or signal. Furthermore, since analysis techniques differs depending on which domain the signal is being analyzed in, spatial and temporal signals can be converted into the frequency domain, or vice-versa, for mathematical convenience or more effective data acquisition. Fourier and Laplace transforms are the functions used for the conversion between these domains.

Frequency domain analysis is performed by considering the individual frequency components of the full range of frequencies that one such signal is comprised of. A useful application for this method is in considering problems like motion blur in images. Since devices such as cameras don't capture an image in an instant, but rather over an exposure time, rapid movements cause the acquired image to have blur that represents one object occupying multiple positions over this exposure time. In a blurred image, edges appear vague and washed out meaning that over those areas their frequency components will be similar. Ideally, the edges would be sharp and that would be reflected by a significant frequency difference along those edges. This project explores the efficiency of using frequency domain techniques to remove motion blur from images.

The overall approach consists of taking an image, converting it into its spatial frequencies, developing a point spread function (PSF) to filter the image with, and then converting the filtered result back into the spatial domain to see if blur was removed. This was performed in



several steps, each of which built from having a greater understanding of the one preceding it. The first step was taking a normal (i.e. not blurred) image, creating a known blurring PSF, and then filtering the image so as to add blur to it. The next step was removing this blur by various methods, but with the information about the PSF that was used to create the blur. After that, de-blurring was performed without knowing anything about nature of the blurring PSF, except for its size. Finally, an algorithm was developed for removing blur from an already blurry image with no information regarding the blurring PSF.

4.2 BLURRING

The first process that was performed creating a point spread function to add blur to an image. The blur was implemented by first creating a PSF filter in matlab that would approximate linear motion blur. This PSF was then convolved with the original image to produce the blurred image. Convolution is a mathematical process by which a signal, in this case the image, is acted on by a system, the filter, in order to find the resulting signal. The amount of blur added to the original image depended on two parameters of the PSF: length of blur (in pixels), and the angle of the blur. These attributes were altered to generate different amounts of blur, but ultimately a length of 31 pixels and an angle of 11 degrees were found to add sufficient motion blur to the image.

4.3 KNOWN PSF DEBLURRING

After a known amount of blur was introduced into the image, an attempt was made to restore the now blurred image to its original form. This was done using several algorithms. In our treatment, a blurred image, i results from:

$$i(x) = s(x) \otimes o(x) + n(x)$$

Where s is the point spread function and is convolved with the 'perfect' image o. In addition, some additive noise, n, may be present. The de-blurring algorithms used here (Lucy-Richardson, Wiener, and regularized), all attempt to get o, from the equation above by means of deconvolution.

The Weiner filter is an inverse filter that employs a linear deconvolution method.

Linear deconvolution means that the output (o) is a linear combination of the input. With an inverse filter we imagine that there exists inverse Fourier transform of a transfer function y(x) such that

$$o(x) = y(x) \otimes i(x)$$

We may change the 1st equation to the frequency domain by using a Fourier transform. Neglecting noise, we have:

$$I(w) = \tau(w)O(w)$$

Now the image may have a band limit Ω . In this case it is not good to work close to this limit. It has been found that the optimum band width, Ωp is given by

$$\Omega_p = \Omega \left[1 - \sqrt{\frac{\Phi_x}{\Phi_{\Phi}}} \right]$$
 Where

F $_{\rm o}$ and f $_{\rm n}$ are the power spectra of the object and noise, respectively. Given all of this, Norbert Wiener found the optimum transfer function to be:

So, for the restored image, we have because the Weiner filter is a linear filter, it is computationally less intensive but it also gives poorer results when noise is introduced. Higher quality filters, such as the Lucy-Richardson, are non-linear.



The Lucy-Richardson algorithm is derived from counting statistics by means of maximizing the likelihood of the solution. The image i is given by

 $i(x) = o(x) \otimes s(x)$

5. RESULT

Two main approaches were used to evaluate the results of the aforementioned procedures. The first was a simple qualitative measure of blur removal. A known amount of blur, but no noise, was added to an image, and then the image was filtered to remove this known amount of blur using Wiener, regularized and Lucy-Richardson deblurring methods. The regularized and Wiener techniques produced what appeared to be the best results. They were largely able to restore the image to its original form, although it was grainier. This grainy effect was especially prevalent in regions that had been low in contrast prior to the initial blurring. We believe this to be due to the fact that factor smeared relatively in low contrast regions the blur similar tones into fewer, indistinguishable ones that could not be perfectly restored. It was surprising that the Lucy-Richardson method produced the worst results in this instance, as it is a non-linear technique, and supposedly more advanced. However, after Gaussian noise was added to the image in addition to blur, the Lucy-Richardson algorithm actually performed the best.

This context help make sense of the previous problem because when just blur is added, only a linear modification is being made and so the linear Wiener restoration technique should work the best. Introducing Gaussian noise, and thus a degree of spatial non linearity, caused the nonlinear Lucy-Richardson method to produce the best results.

As information about the PSF that was used to perform the blurring was removed from the algorithms, the efficacy of blur removal dropped. In the blind deblurring method, the majority of the blur itself was removed, but a lot of the image's original detail was lost and a "ringing" effect could be seen across the entire image. The ringing was a result of using a PSF designed to remove blur in areas of high contrast (edges, where blur should be most prominent) over the entire image, and thus creating high contrast in waves across the image. The image quality was vastly improved when the edge checking function was implemented so that only true edges would receive this edge deblurring treatment.

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While qualitative analysis is a good first step for determining whether an image processing technique has succeeded or not, quantification of the results is necessary. Quantification allows for a more exact measurement of improvement, and more importantly, allows for comparison between the efficacies of different methods. In this instance, quantification was performed by finding the mean pixel intensity value over a region, the standard deviation of the pixel values over that same region, and then determining the image contrast ratio; the ratio of standard deviation to the mean. This ratio normalizes the standard deviation so that any changes to the mean intensity caused by our filtering technique would not influence determination of filter quality, and acts as a direct measure of image contrast. Contrast ratio for a restored image should be higher than that of the original blurred image. The reason for this is that blurring an image causes the pixels surrounding a moving edge, the area where motion blur occurs, to become washed out and all take on similar intensity values. This leads to a low standard deviation relative to the mean. Once the image is filtered, however, contrast between the edge and the object should be restored and this contrast will cause a higher value for the standard deviation to mean ratio. Comparisons between blurred and filtered images were made using a parameter of percent improvement

Image	Blurred Ratio	Filtered Ratio	Improvement (%)
Woman	0.0975	0.0986	1.13%
Train	0.0737	0.0746	1.22%
Fish	0.1027	0.1044	1.66%
Table 1: Deblurring	Efficacy for Three Full	Images	
Pixel Contrast Meas	surement for Blurred	and Filtered Image S	egments
Image	Blurred Ratio	Filtered Ratio	Improvement (%)
Woman Segment	0.0323	0.0370	14.55%
Train Segment	0.0520	0.0573	10.19%
Fish Segment Table 2: Deblurring	0.0517 Efficacy for Three Imag surement for Various	0.0630 ge Segments Techniques (No Nois	21.86% e Added)
Fish Segment Table 2: Deblurring Pixel Contrast Meas Technique	0.0517 Efficacy for Three Imag surement for Various Blurred Ratio	0.0630 ge Segments Techniques (No Nois Filtered Ratio	21.86% e Added) Improvement (%)
Fish Segment Table 2: Deblurring : Pixel Contrast Meas Technique Wiener	0.0517 Efficacy for Three Imag surement for Various Blurred Ratio 0.0517	0.0630 ge Segments Techniques (No Nois Filtered Ratio 0.0606	21.86% e Added) Improvement (%) 17.21%
Fish Segment Table 2: Deblurring : Pixel Contrast Meas Technique Wiener Regularized	0.0517 Efficacy for Three Imag surement for Various Blurred Ratio 0.0517 0.0517	0.0630 ge Segments Techniques (No Nois Filtered Ratio 0.0606 0.0606	21.86% e Added) Improvement (%) 17.21% 17.21%
Fish Segment Table 2: Deblurring Pixel Contrast Meas Technique Wiener Regularized Lucy-Richardson	0.0517 Efficacy for Three Imag surement for Various 0.0517 0.0517 0.0517	0.0630 ge Segments Techniques (No Nois Filtered Ratio 0.0606 0.0606 0.0563	21.86% e Added) Improvement (%) 17.21% 8.90 %
Fish Segment Table 2: Deblurring . Pixel Contrast Meas Technique Wiener Regularized Lucy-Richardson Table 3: Deblurring .	0.0517 Efficacy for Three Imag surement for Various 0.0517 0.0517 0.0517 Efficacy for Three Filte	0.0630 ge Segments Techniques (No Nois Filtered Ratio 0.0606 0.0606 0.0563 vring Techniques	21.86% e Added) Improvement (%) 17.21% 17.21% 8.90 %
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Fish Segment Table 2: Deblurring . Pixel Contrast Meas Technique Wiener Regularized Lucy-Richardson Table 3: Deblurring . Pixel Contrast Meas Technique Wiener	0.0517 Efficacy for Three Imag surement for Various Blurred Ratio 0.0517 0.0517 Efficacy for Three Filte surement for Various Blurred Ratio 0.0507	0.0630 ge Segments Techniques (No Nois Filtered Ratio 0.0606 0.0563 vring Techniques Techniques (Noise Au Filtered Ratio 0.0692	21.86% e Added) Improvement (%) 17.21% 8.90% dded) Improvement (%) 36.49%
Fish Segment Table 2: Deblurring . Pixel Contrast Meas Technique Wiener Regularized Lucy-Richardson Table 3: Deblurring . Pixel Contrast Meas Technique Wiener Regularized	0.0517 Efficacy for Three Imag surement for Various 0.0517 0.0517 Efficacy for Three Filte surement for Various Blurred Ratio 0.0507 0.0507	0.0630 ge Segments Techniques (No Nois Filtered Ratio 0.0606 0.0563 vring Techniques Techniques (Noise Ar Filtered Ratio 0.0692 0.0691	21.86% e Added) Improvement (%) 17.21% 17.21% 8.90 % dded) Improvement (%) 36.49% 36.29%



Fig a. image with no noise



Fig b. image with gaussian noise

According to Table 1, the technique we developed for blind filtering increased the contrast ratio across each of the images that it was used on 6 by a little more than 1% each. While this does suggest that the overall contrast of each image was improved, it is not the most meaningful way to consider the data. Since blur does not occur uniformly across an entire image, but rather most significantly along moving edges, the most appropriate way to measure the success of our method is to find the contrast ratio across a severely blurred region. This was accomplished by isolating a highly blurred subset of the original image, and comparing it to the same sub set in the filtered image. Table2 shows that over regions of high blur, the contrast ratio for the filtered image is ten to twenty percent greater than in the original blurred image – a significant increase. The high contrast ratio came from an image (Fish) with computer added blur while the lower contrast ratios were found in images (Woman and Train) with natural motion blur due to movement during image acquisition exposure time. Because the computer generated blur was more severe than natural blur, it makes sense that a greater degree of contrast restoration occurred in the image that was blurred computationally.

Finally, comparisons were drawn between the regularized, Lucy-Richardson, and Wiener filtering methods for an image with computationally generated blur under both no noise added, and Gaussian noise added conditions. Table 3 confirms the qualitative observations that were made, demonstrating that Weiner and regularized techniques show almost twice as much improvement over the blurred image than Lucy-Richardson does. The results shown in Table 4, the efficacy of these techniques when noise is added, appear odd at first glance. The extremely high values for Wiener and regularized



techniques are actually a result of their inability to filter out the noise across the region of interest. This makes the contrast appear much higher than it should be for a "well processed" image. The lower, but still reasonable, contrast value found in the Lucy Richardson method in this instance actually represents that method's superiority for instances in which noise is added.

6. CONCLUSIONS

Through this paper, several techniques for frequency domain image processing were explored. Both presented here deblurring methods lend themselves to the image reconstruction. In the simplest of these, motion blur was added to a deblurred image. In the most advanced, blur was filtered out of a partially blurred image when no information regarding the blurring PSF was known. This was accomplished by optimizing an edge detection algorithm, finding how to set appropriate thresholds for restoring blurred out PSFs, and discerning how many filtering iterations were necessary to remove the blur. Ultimately improvements on the order of ten to twenty percent were obtained.

References

- [1] BOVIK. A.: Handbook of Image and Video Processing, Academic Press, 2002
- [2] GONZALES. R, WOODS. R.: Digital Image Processing, Addison Wesley, 1993
- [3] HANSEN. P.: Deconvolution and regularization with Toeplitz matrices, Informatics and Mathematical Modeling, Danish Technical University, Numerical Algorithms 29, 2002
- [4] GROETSCH. G.: Inverse Problems in the Mathematical Sciences, Vieweg Mathematics For Scientists and Engineers, Vieweg, 1993
- [5] BERTERO. M, BOCCACCI. P.: Introduction to Inverse Problems in Imaging, IOP Publishing Ltd London, 1998
- [6] HANSEN. P.: Regularization tools: A Matlab package for the analysis and solution of discrete ill posed problems, Numerical Algorithms vol 6, 1994
- [7] KAMM. J, NAGY. J.: Kronecker product and SVD approximation in image restoration, Linear Algebra Applications, 1998
- [8] Ayata et al. Laser Speckle Flowmetry for the Study of Cerebro vascular Physiology in Normal and Is chemi Mouse Cortex. Cerebral Blood Flow Metabolism. 2004;24(7):744-755 Podilchuk, C. Preprocessing for Enhanced.
- [9] P. Bojarcz ak, Z. Łukasik Image Deblurring Wiener Filter Versus Tsvd Approach, Advances in Electrical and Electronic Engineering pg .86-89
- [10]M. Chang, A. Tekalp, and A. Erdem. Blur identication using the bispectrum. IEEE Transactions on Signal Processing, 39(10):2323 2325, 1991.
- [11] D. Kundur and D. Hatzinakos. A novel blind deconvolution scheme for image restoration using recursive filtering. IEEE Tran. on Signal Processing, 46(2):375 390,1998.
- [12] R.Lagendijk, J. Biemond, and D. Boekee. Identification and restoration of noisy blurred images using the expectation maximization algorithm. IEEE Transactions on Acoustics, Speech, and Signal Processing, 38(7):1180 1191, 1990.
- [13]R. Lagendijk, A. Tekalp, and J. Biemond. Maximum likelihood image and blur identication: a unifying approach. Opt. Eng., 29(5):422 435, 1990.
- [14]R. Lagendijk, A. Tekalp, and J. Biemond. Maximum likelihood image and blur identication: a unifying approach. IEEE Sig. Processing Magazine, 13(3):43 64, 1996